

A Quasi-maximal mixing ansatz for neutrino oscillations.

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Abstract

Inspired by the atmospheric multi-GeV neutrino data, we consider neutrino flavor mixing matrices which are maximal in a 2×2 subsector. This condition is a strong restriction: the full matrix including complex phases depends essentially on one parameter. The survival probability is an universal function of L/E , independent of generation, in the region of interest for the accelerator and atmospheric experiments. It is possible with the solar neutrino data alone to recover completely the mixing matrix suggested by the atmospheric data. The results are not essentially modified by the MSW effect in the solar data. Consequences for future experiments are considered.

The hypothesis that the mixing in the Lepton sector is threefold maximal has been discussed recently [1]. Such mixing would imply specific forms for the Lepton mass matrices, corresponding to a cyclic permutation symmetry among generations. The mixing is maximal in the sense that all the elements of the mixing matrix have equal modulus. The intrinsic CP violation is also maximal. As a consequence of this hypothesis the survival probabilities, as measured in disappearance experiments, $P_{ll} = P(\nu_l \rightarrow \nu_l)$ are identical. $P = P_{ll}$ is an universal function of L/E , L the distance of propagation, E the neutrino energy. In this model detailed predictions depend on the character of the neutrino squared mass difference spectrum, a pronounced hierarchy must be assumed in order to accommodate the experimental data, particularly the solar data.

In [2] and [3] it is shown that matter effects do not contribute to the averaged values of the probabilities of transition of solar neutrinos into other states in case of maximal mixing of any number of species.

Completely "democratic" mass matrices has been proposed already in the context of the Seesaw mechanism (see [4] and references therein). As it can be easily shown maximal mixing appears as a possible particular case in diagonalizing democratic mass matrices.

There are by now only two experimental groups which claim to have positive evidence of neutrino oscillations, KAMIOKANDE [5] with its multi-GeV data set and the LSND experiment [6].

A new analysis of the published KAMIOKANDE data of multi-GeV atmospheric neutrinos is presented in terms of three flavors in [7]. The optimum set of parameters is found to be $(\Delta m_{21}^2, \Delta m_{31}^2) = (3.8 \times 10^{-2}, 1.4 \times 10^{-2}) eV^2$, and $(\theta_{12}, \theta_{13}, \theta_{23}) = (19^\circ, 43^\circ, 41^\circ)$. θ_{ij} are the mixing angles in the standard 3 flavor parameterization [8]. The explicit mixing matrix corresponding to these angles is:

$$u_{multi-GeV} = \begin{pmatrix} 0.69 & 0.23 & 0.68 \\ -0.67 & 0.57 & 0.48 \\ -0.27 & -0.79 & 0.55 \end{pmatrix} \quad (1)$$

In [7], the minimization is carried out over a sparse grid of the parameter space by CPU needs. Possible complex phases has been neglected. A maximal mixing model with the standard mass hierarchy is hardly compatible with this result even in a broad sense.

The LSND experiment observes $\bar{\nu}_e$ in excess from a proton beam dump at LAMPF. It corresponds to a oscillation probability of $(3.1 \pm 1.0 \pm 0.5) \times 10^{-3}$ [6], if the observed excess is interpreted as a $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transition.

In Fig.(1), we summarize virtually all the experimental limits available until now about neutrino oscillations. A main observation can be drawn. There are fundamentally only two differentiated regions in the L/E line. This implies there is only one relevant mass difference scale ($\Delta m_0^2 \approx 10^{-1} - 10^{-2} eV^2$). There are only two possibilities then: a) two of the generations are degenerated or nearly degenerated ($\Delta m_1^2 < 10^{-11} eV^2$); b) the mass differences between all the generations are approximately equal ($\approx \Delta m_0^2$).

For low L/E ($L/E \ll 1/\Delta m_0^2$), the survival probabilities are universal $P_{ee} \simeq P_{\mu\mu}$. At high L/E , P_{ee} is practically constant within experimental errors. This is specially true if one discard the Homestake experiment (or attribute a bigger error to it).

The interpretation of the atmospheric data presents special problems. In one side, it is not possible to consider only purely survival probabilities in general for three neutrino species. In another side, the spread in L and E are significantly larger and less well-known than in the rest of experiments. A detailed analysis for each experiment is needed, out of the scope of this work.

Much less data is available for transition probabilities, which are shown in Fig.(1) (bottom).

The object of this work is to show how we can recover a mixing matrix equivalent to (1), including in addition complex phases, from the information contained in Figure (1) together with a highly predictive mixing matrix ansatz.

We are interested in the case where the mixing is maximal in the $\mu - \tau$ sector, in the sense that all the matrix entries u_{ij} , ($2 \leq i, j \leq 3$), are of equal magnitude. The cases where the mixing is maximal in the $e - \mu$ or $e - \tau$ sectors are completely analogous and will be treated later. Explicitly, we consider the matrix ansatz

$$u = \frac{1}{k} \begin{pmatrix} \alpha & \beta & \gamma \\ \epsilon & 1 & \exp -i\lambda_1 \\ \delta & \exp -i\lambda_2 & \exp -i\lambda_3 \end{pmatrix} \quad (2)$$

By unitarity the values of the different elements are related to the phases λ_i (it is necessary to suppose $\beta \neq 0$):

$$\begin{aligned} |\beta|^2 = |\gamma|^2 = |\epsilon|^2 = |\delta|^2 &= 2 \left| \cos \frac{\Lambda}{2} \right| \\ \kappa = |k|^2 &= 2 + |\beta|^2 = 4 \sin^2 \frac{\Lambda}{4} \\ |\alpha|^2 &= 2 - |\beta|^2 = 4 \cos^2 \frac{\Lambda}{4} \\ \arg(\gamma - \beta) &= \frac{\Lambda_+}{2}; \quad \arg(\delta - \epsilon) = \frac{-\Lambda_-}{2} \\ \cos(\Delta_\delta) &= \pm \cos \frac{\Lambda}{4} = \pm \frac{|\alpha|}{2} \\ \Lambda = \lambda_1 + \lambda_2 - \lambda_3; \quad \Lambda_+ &= \lambda_1 - \lambda_2 + \lambda_3; \quad \Lambda_- = \lambda_1 - \lambda_2 - \lambda_3 \\ \Delta_\delta &= \arg(\beta + \epsilon - \alpha) \\ |a_{CP}| \equiv 2 \frac{|\Im(u_{11}u_{22}u_{12}^*u_{21}^*)|}{|u_{11}u_{22}|^2 + |u_{12}u_{21}|^2} &= \frac{1}{4} |\alpha| |k| \end{aligned}$$

The convention independent parameter a_{CP} [9] gives a measure of the intrinsic CP violation. Note that as long as two of the three masses are degenerate or nearly degenerate the effective CP violation is in any case null or very small.

For the case $\beta = 0$, the mixing matrix has the trivial form

$$u = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} \exp i\delta & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix} \quad (3)$$

There is a decoupling between the first and the other generations. The mixing between the second and the third one is rigorously maximal. This case, which it is not supported apparently by experimental evidence will appear later when we consider $e - \mu$ maximal mixing.

The transition probabilities between weak states can be written as

$$P_{\alpha\beta} = | \text{tr} \exp -iH_0 t W_{(\alpha\beta)}^\dagger |^2 \quad (4)$$

where H_0 is taken diagonal and the set of matrices $W_{(\alpha\beta)}$ is defined by

$$W_{ij,(\alpha\beta)} = u_{\alpha i}^* u_{\beta j} \quad (5)$$

The probabilities $P_{\alpha\beta}$ are linear combinations of c_{ij} , $s_{ij} = \cos, \sin(2.57\Delta m_{ij}^2 L/E)$, $\Delta m_{ij}^2 = m_i^2 - m_j^2$ and the neutrino masses m_i (eV), L (m), E (MeV).

From the matrix given by (2), we compute the survival probabilities:

$$P_{ee} = P_{ee}^\infty - 2 \frac{\kappa^2 - 6\kappa + 8}{\kappa^2} (c_{12} + c_{13}) + 2 \frac{\kappa^2 - 4\kappa + 4}{\kappa^2} c_{23} \quad (6)$$

$$P_{\tau\tau} = P_{\mu\mu} = P_{\mu\mu}^\infty + 2 \frac{\kappa - 2}{\kappa^2} (c_{12} + c_{13}) + \frac{2}{\kappa} c_{23} \quad (7)$$

with

$$P_{ee}^\infty = \frac{1}{|k|^4} (|\alpha|^4 + |\beta|^4 + |\gamma|^4) = 3 - \frac{16}{\kappa} + \frac{24}{\kappa^2} \quad (8)$$

$$P_{\tau\tau}^\infty = P_{\mu\mu}^\infty = \frac{1}{|k|^4} (2 + |\beta|^4) = 1 - \frac{4}{\kappa} + \frac{6}{\kappa^2} \quad (9)$$

With the same mixing matrix, some of the transition probabilities are:

$$\begin{aligned} P_{\bar{\mu} \bar{e}, \mu e} &= \frac{\kappa - 2}{\kappa^2} \left(|\alpha|^2 + 2 + 2 |\alpha| (\cos(E_{12} \pm \Delta_\delta) + \cos(E_{13} \pm \Delta_\delta \pm \Lambda/2)) + 2 \cos(E_{23} + \Lambda/2) \right) \\ P_{\mu\tau} &= \frac{1}{\kappa^2} \left(\kappa + 2 |\beta|^2 (\cos(E_{12} - \Delta/2) + \cos(E_{13} + \Delta/2)) + 2 \cos(E_{23} + \Delta) \right) \end{aligned} \quad (10)$$

In the second formula the $+$ ($-$) sign corresponds to $\nu_\mu \nu_e (\bar{\nu}_\mu \bar{\nu}_e)$ transitions. $E_{ij} = \Delta m_{ij}^2$. We note that all the $P_{\alpha\beta}$ depend only on one parameter (κ or Λ).

For the interpretation of the data of any concrete experiment, it is necessary to average the c_{ij} , s_{ij} 's taking into consideration the finite volume of the detector; the space, time and energy resolutions; the energy dependent cross sections and experimental detection efficiencies. Unfortunately the different experimental groups publish little or unmanageable information about these details (see [10] for an exception, where the averaging weight function is given in an explicit and practical way).

The effect of the averaging can be approximated conveniently considering an uniform distribution of the ratio L/E in the range $[0, 2L/E]$. This is the prescription adopted in [1]. We have examined many other prescriptions, in particular eliminating the $L/E \rightarrow 0$

region. We have checked that the following results are independent of the averaging procedure which is adopted.

In the region of very high L/E , as in the solar neutrino case for not so small Δm^2 's, the averaged c_{ij}, s_{ij} are always practically zero. The transition probabilities become independent of L/E there. The statistical average of the values from the four solar neutrino experiments (taken from Table (1) in [1]) is $P_{ee} = 0.52 \pm 0.04$. In Fig.(2), we plot P_{ee}^∞ as a function of $\kappa = |k|^2$. we see that there is only a very restricted region in the κ line allowed by the solar neutrino data. If we request $P_{ee} \simeq 1/2$, then there are two possible solutions $\kappa = 2.4$ or $\kappa = 4$. If $P_{ee} \simeq 0.4 - 0.5$ then $\kappa \simeq 2.4 - 2.5$ or $\kappa \simeq 3.8 - 4$. If P_{ee} is clearly greater than $1/2$ but less than ≈ 0.6 then only one region is allowed: $\kappa \simeq 2.3 - 2.4$. The threefold maximal mixing correspond to the value $\kappa = 3$. This case is discarded by the solar data except if a particular strong mass hierarchy is considered as done in [1].

For definiteness, we take $\kappa = 2.5$, in this case

$$\Lambda = 3.6; \Lambda_\delta = 2.2; \quad |\alpha|^2 = 1.5; \quad |\beta|^2 = 0.5$$

The mixing matrix is

$$u = \begin{pmatrix} 0.77e^{i\delta} & 0.44 & 0.44e^{-i\lambda/2} \\ 0.44 & 1 & 1 \\ 0.44e^{i\lambda/2} & e^{-i\lambda} & 1 \end{pmatrix} \quad (11)$$

$$\lambda = 3.65 \text{ rad}, \delta = 2.23 \text{ rad}$$

The allowed high κ region is only possible if $P_{ee}^\infty \simeq 0.4 - 0.5$. In the limit $\kappa \rightarrow 4$, we have $|\alpha| = 0$. The mixing matrix takes the form ($\lambda_3 = 0$ for convenience):

$$u = \frac{1}{2} \begin{pmatrix} 0 & w & -sw \\ w & 1 & s1 \\ -sw & s1 & 1 \end{pmatrix} \quad (12)$$

$$w = \sqrt{2} \exp -i\delta \quad s = \pm 1$$

There are not CP violation, $a_{CP} = 0$.

We give explicitly the form of some of the matrices $W_{(\alpha\beta)}$ in this case:

$$W_{(11)} = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \quad (13)$$

$$W_{(22)} = \frac{1}{4} \begin{pmatrix} 2 & w^* & sw^* \\ w & 1 & s1 \\ sw & s1 & 1 \end{pmatrix} \quad W_{(33)} = \frac{1}{4} \begin{pmatrix} 2 & -w^* & -sw^* \\ -w & 1 & s1 \\ -sw & s1 & 1 \end{pmatrix}$$

The probabilities become:

$$P_{ee} = \frac{1}{2}(1 + c_{23}) \quad (14)$$

$$P_{\mu\mu} = P_{\tau\tau} = \frac{1}{8}(3 + 2c_{21} + 2c_{31} + c_{32}) \quad (15)$$

$$P_{\mu e} = P_{\bar{\mu}e} = \frac{1}{4}(1 - c_{23}) \quad (16)$$

$$P_{\mu\tau} = \frac{1}{8}(3 - 2c_{21} - 2c_{31} + c_{32}) \quad (17)$$

The maximal mixing in the $e - \mu$ or $e - \tau$ sectors. These cases are obtained trivially from the previous $\mu - \tau$ case. P_{ee} becomes equal to the $P_{\mu\mu}$ which was obtained before. According to Figure (2) (now to the curve labeled as $P_{\mu\mu}$) this case is incompatible with a large value for P_{ee} ($> \approx 0.50$). If we force a value for $P_{ee} \approx 0.4$ (and subsequently we favour the Homestake data) then $\kappa \approx 2$ and $|\beta|^2 \approx 0$, we recover approximately the matrix given by Eq.(3).

Including matter effects. It is important to show that matter effects does not affect the solar probabilities, as they play an important role in our derivation. We can easily show that they effectively doesn't affect for the case given by the mixing matrix (12). It was shown the same in [3] for the fully maximal case. We expect for the general case the effect, if any, to be small also. The amplitude matrix in an arbitrary basis can be written with generality as

$$\mathcal{A} = \exp -iH_0 t A_r \quad (18)$$

where the matrix A_r accounts for any matter effect. The transition probabilities between weak states can be written as

$$P_{\alpha\beta} = | \text{tr} \exp -iH_0 t A_r W_{(\alpha\beta)}^\dagger |^2 \quad (19)$$

with $W_{(\alpha\beta)}$ the same matrices as before.

For the solar case A_r can be further decomposed (see [11, 12]) as

$$A_{ij,r} = \delta_{ij} + W_{ij,(11)} A_{ij,s} \quad (20)$$

For the matrix (11), the matrix $W_{(11)}$ is given by Eq.(13). So A_r has the form

$$A_r = \left[\begin{array}{c|c} 1 & 0 \\ \hline 0 & A_r^{(2)} \end{array} \right] \quad (21)$$

and

$$A_r \times W_{(11)} = \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & A_r^{(2)} \times W_{(11)}^{(2)} \end{array} \right] \quad (22)$$

where the superindex indicates the restriction to a dim 2 matrix.

$$P_{ee} = | \text{tr} e^{-iH_0 t} A_r^{(2)} \times W_{(11)}^{(2)} |^2 \quad (23)$$

We have reduced the problem to a problem with two generations and maximal mixing between them. As it can be seen directly from Eq.(36) in [11], the probability in this case is independent of any value of the other parameters.

In Fig.(1) (top, lines) we plot $P_{ee}, P_{\mu\mu}$ for different cases (for any case $P_{\mu\mu}$ is the lowest line), always with the mass spectrum given by [7] (except case D). We see there are not strong differences among the different cases. The case D, where a different mass spectrum has been used, is clearly not satisfactory in the medium range. Accordingly, in Fig.(1) (bottom), we plot transition probabilities. The model predicts the general trend of the data. The LSND point would situated itself in the transition region given by the mass scale Δm_0^2 . However the low error quoted in the last published result makes very difficult to reconcile model and experiment.

Conclusions and consequences for future experiments. We have seen that quasi-maximal mixing in the $\mu - \tau$ sector represents a simple, highly predictive ansatz to describe the general trend of all the experimental evidence with three generation neutrino oscillations. Small local deviations, for example among the solar data, could be accounted using additional mechanisms: residual matter oscillation effects, magnetic field effects. The introduction of a non-unitary, still quasi-maximal, mixing matrix should deserve further attention in order, for example, to understand better the LSND value.

For the CHORUS and NOMAD accelerator experiments [15, 16] the value of the quotient $L/E \approx 10^{-2}$. According to the scenario made plausible by this work they fall in a region where very little signal ($P_{\mu\tau} \approx 0$) is expected. The results (soon coming) of these experiments would allow to improve the existing exclusion plots but they would have a very little discovering potential. That situation could improve if they were able to increase significantly their efficiency for the lower energy neutrinos.

During the period 1996-1997 two new solar experiments are scheduled to start, Super-Kamioka and SNO ([17]). Both experiments will be able to measure the flux of solar neutrinos coming from the 8B decay as a function of the neutrino energy. Any departure from the theoretically well known flux could be a signal for neutrino oscillations. As presented in this work, P_{ee} is constant at large L/E . For a fixed energy, the average over L alone would be also enough to warranty the constancy of P_{ee} , at least for appropriate long term measurements, and the neutrino energy spectrum could appear unmodified even in presence of neutrino oscillations. However, as it is shown in [2], still the neutrino oscillations could have checkable visible consequences.

Long baseline experiments as Fermilab/Soudan or CERN/Gran Sasso with $L/E \approx 10^2 - 10^4$ offer the most realistic expectation of being able to find positive evidence of neutrino oscillations. In the same sense, the upgrading of LSND increasing L in a factor ≈ 10 would be highly recomendable. Taking into account the Fig.(1) they would enter in a region with with a discovering potential nearly guaranteed. The rapid increase of the transition probability here could make innecesary even further improvements in the collimation of its neutrino beam.

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Experiment	L/E (m/MeV)	P(90%)	Ref.
KARMEN	17.7 m/40 MeV = 0.4	$P_{\bar{\nu}_\mu \bar{\nu}_e} < 3.1 \cdot 10^{-3}$	[13]
BNL	1 km/1-4 GeV = 0.2 – 1	$P_{\mu e}, P_{\bar{\mu} \bar{e}} < 1.5 \cdot 10^{-3}$	[14]
LSND	30 m/30-60 MeV = 0.5 – 1	$P_{\bar{\mu} \bar{e}} = 3.1 \pm 1.0 \pm 0.5 \cdot 10^{-3}$	[6]
E531	$\approx 0.01 - 0.04$	$P_{\nu_\mu \nu_\tau} < 2 \cdot 10^{-3}$	[10]
CHORUS/NOMAD	0.8 km/30 GeV = 0.03	$P_{\mu\tau}$	[15, 16]

Table 1: Results for the transition probabilities measured in diverse reactor experiments.(CHORUS and NOMAD have not presented results so far).

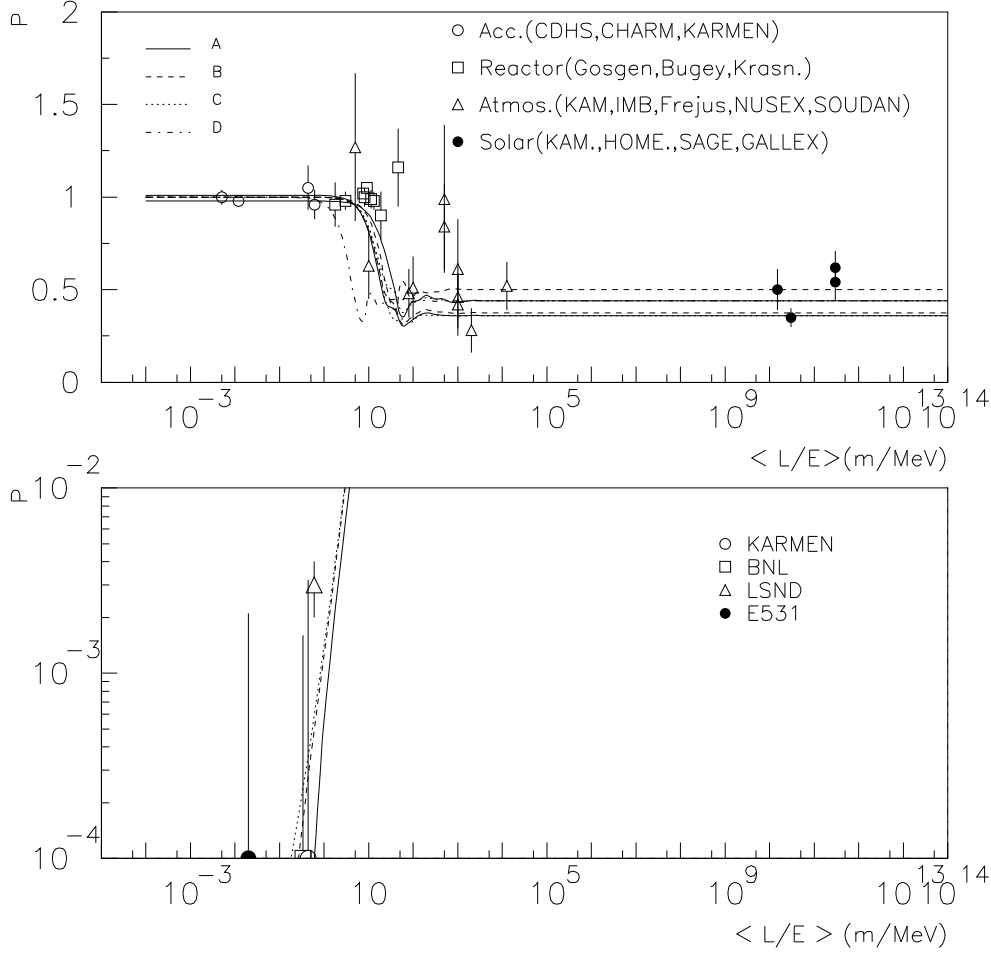


Figure 1: Top Figure. Individual data points: P_l measured in diverse experiments. See Table (1) in [1] for detailed information about very individual point. The atmospheric data points are shown with no corrections. Continuous and dashed lines: $P_{ee}^\infty, P_{\mu\mu}^\infty$ for different mixing matrices, the mass differences are the ones obtained in [7] except for (D). (A) With the mixing matrix (1). (B) With the matrix (12). (C) With the matrix (11). (D) With the matrix (11) and $(\Delta m_{21}^2, \Delta m_{31}^2) = (1.38 \times 10^{-1}, 1.14 \times 10^{-1}) eV^2$. Bottom Figure. Transition probabilities coming from different experiments (see Table (1)). The different lines correspond to the case (C) referred before except the dot-dashed one which correspond to case (D). The cases (A), (B) have been omitted for clarity. Their behavior is essentially the same as case (C).

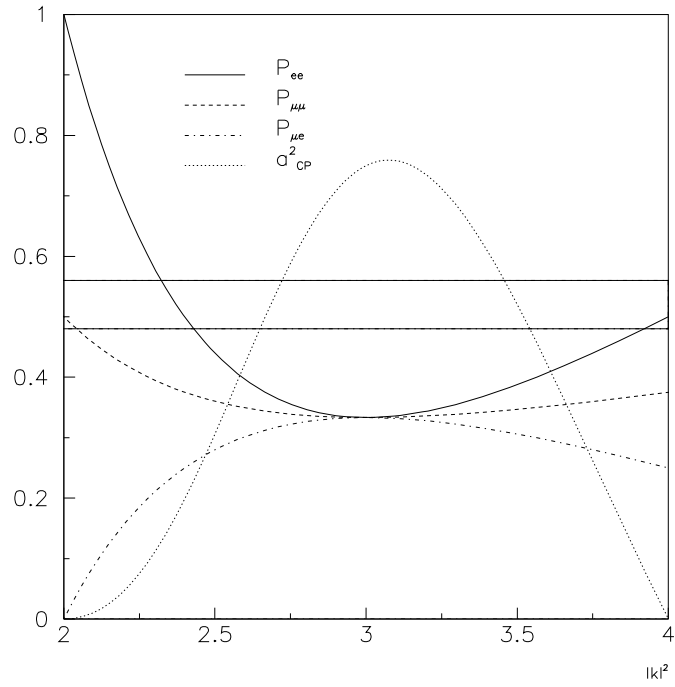


Figure 2: The probabilities $P_{ee}^\infty, P_{\mu\mu}^\infty$ (eqs. (8-9)) and $|a_{CP}|^2$ as a function of $\kappa = |k|^2$. The value $|k|^2 = 3$ correspond to the threefold maximal mixing, here $P_{ee}^\infty = P_{\mu\mu}^\infty$. The horizontal band correspond to the average of the solar neutrino experiments: $P_{ee}^\infty = 0.52 \pm 0.04$.

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